



## Latin Squares World

Latin Squares (LS) are fun and useful, but they can be hard to make.

Examples:

Creating a playing schedule.

	Czech	Ghana	Italy	USA
Czech	–	17 June	22 June	12 June
Ghana	17 June	–	12 June	22 June
Italy	22 June	12 June	–	17 June
USA	12 June	22 June	17 June	–

Sudoku are special types of LS.

5	6	2	7	9	4	1	8	3
4	3	8	1	5	2	7	6	9
9	1	7	6	3	8	5	2	4
6	2	9	3	1	7	8	4	5
3	7	4	5	8	9	2	1	6
8	5	1	4	2	6	9	3	7
2	8	5	9	4	3	6	7	1
1	4	6	8	7	5	3	9	2
7	9	3	2	6	1	4	5	8

A **latin square** of **order  $n$**  is an  $n \times n$  **matrix(array)** in which each of  $n$  symbols occurs exactly once in each row and once in each column.

A **transversal** of a LS is a selection of entries which includes exactly one entry from each row and column and one of each symbol.

Some uses of latin squares:

Scheduling and tournaments, Design and analysis of experiments, Data transmission: Construction of error detecting and correcting codes, Cryptography, Software configuration and testing, Algebra, Graph theory, Finite geometry, Tomography

1. a) Complete the LS below.

	0	1	2	3	4
0	1		2		4
1		2	3	4	
2	2		4		
3	3	4	0	1	2
4	4	0	1	2	3

b) What is the order of this LS?

c) Show a transversal in the LS by circling the entries.

d) How many transversals can you find? Note that the same entry may occur in more than one transversal. Hint: Use different colours.

2. Construct two LS of order 4. In the process discover a **cyclic** LS.

a) What do you notice about transversals in your LS?

b) In each LS, swap 2 rows, then 2 columns and then 2 symbols. What happens to your LS? What happens to a transversal?

0	1	2	0	2	1
1	2	0	1	0	2
2	0	1	2	1	0

These LS are special - they are called **orthogonal mates**. When we overlay them then each ordered pair of symbols occurs exactly once. A LS and an orthogonal mate are called a pair of **Mutually Orthogonal Latin Squares (MOLS)**.

- Construct a pair of MOLS of order 5. Choose your symbols to be numbers, colours, both, or your own creative design. Start with a cyclic LS.
- Does a cyclic LS of order 4 have an orthogonal mate? How can transversals help in such questions? Explain why there is no order 2 MOLS.  
Note that there are MOLS of every order except for order 2 and order 6.
- Complete the LS of order 8 so that it is non-cyclic. Does it have a transversal? Does it have an orthogonal mate?

	0	1	2	3	4	5	6	7
0		1	2	3		5	6	7
1	1	2	3	4	5	6	7	0
2	2	3	4	5	6	7	0	1
3	3	4	5	6	7	0	1	2
4		5	6	7		1	2	3
5	5	6	7	0	1	2	3	4
6	6	7	0	1	2	3	4	5
7	7	0	1	2	3	4	5	6